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A MODEL OF BUILDING INVENTORY[1]

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1. The model demonstrated in this article is originated in the Ph.D. dissertation of the author ("Survival Throuht Change: A Developmental Perspective") Presented to Cornell University,1976.

This study attempts to demonstrate a model designating the transformation of the building profile of urban centers as a function of changing demographic, socio-economic, cultural and technological aspects of urban environment. More specifically, it deals with the processes of historic progression of building stock, namely of the construction, survival and demolition of architectural set up.

The model makes explicit in the form of matrix notation, the distribution and changes in the composition of the building stock over time. The processes of historic progression of building inventory which are represented as matrix operations are production, survival and demolition. Different survival coefficients enter into this process in which each rate becomes the by-product of different sets of variables such as the level of technology, socio-economic condition, demographic structure, material use, age, etc. The part of the study which involves the deliniation of relationship between the built environment and the state of urban development is not elaborated here and only the exploratory model of building inventory is presented.

AN INVENTORY MODEL: A STRUCTURAL FRAMEWORK FOR THE ANALYSIS OF CHANGES IN THE PROFILE OF ARCHITECTURAL STOCK

The dominant strand of design activity in urban centers is that of the inescapable change in the physical composition of the city. This physical composition can be described at any time as a combination of newly produced items and of surviving ones. Basically, this represents the volume of physical structures that the city has inherited from previous time periods. The profile of the city's architectural inventory has two dimensions: a size dimension and a time dimension. Size refers to the number of structures, including historic buildings, that make up the building stock of the city in a given time period, while time dimension refers to various periods in which the inherited building stock has been constructed.

The size and time composition of building inventory can be systematized in the form of an accounting model. The elements of this accounting system are buildings. Buildings enter into the stock by construction and leave it by demolition. A time-continuum analysis of building activity

-construction, survival and demolition- can bi developed by describing at various times the possible configurations of the building inventory.

The process described above can be presented as a more generalized analytical one which reflects the changes in the city's building stock. Consider the situation depicted in Figure 1. The initial building stock at time T_1 is the number of buildings N_1 built during the first time period (T_0-T_1). This is a function of different variables such as location, resources, technology, institutions, population,etc.

A certain percentage of this building stock is demolished during the second time period (T_1-T_2), most probably due to construction failure, natural disasters, etc. so that the survival stock at time T_2, at the end of the second time period (T_1-T_2) is,

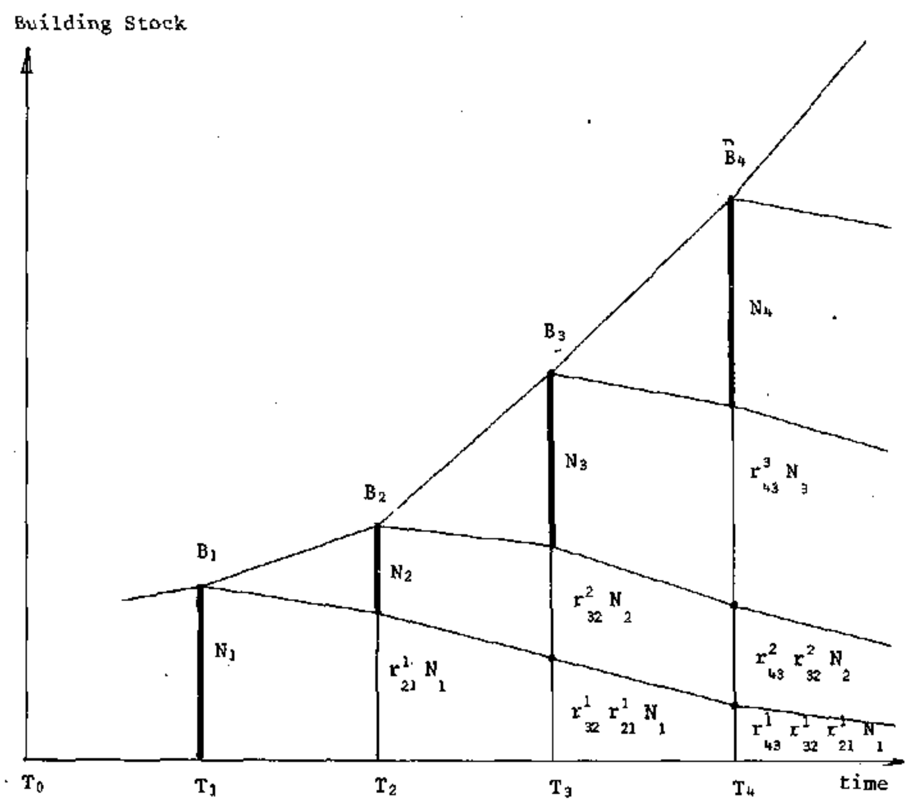

fig. 1 Changes in the Building Stock over time.

$$S_2 = r^1_{21} N_1 \qquad (1)$$

Here, r^1_{21} is the survival rate during the time period two (indicated by the subscripts 2,1) of the buildings built in the first time period (indicated by the superscript 1). The survival rate is obtained by subtracting the rate of demolishment from unity.

The building stock at time T_2, at the end of the second time period (T_1-T_2) is composed of the buildings built during the second time period N_2 and the surviving stock from the first time period, i.e.,

$$B_2 = r^1_{21} N_1 + N_2 . \qquad (2)$$

Here the new building stock N_2 is again a function of a set of variables such as changes in population, technology, institutions, etc. during the second time period. The survival rate r^1_{21} of buildings built in the previous period is determined by a set variables such as changes in population, bielding stock, technology, institutions, income levels, culture, occupancy, age maintenance and repair, deterioration, etc.

Applying the same procedure for succeeding time intervals, we have

$$B_3 = r^1_{32} \; r^1_{21} \; N_1 + r^2_{32} \; N_2 + N_3 \qquad (3)$$

$$B_4 = r^1_{43} \; r^1_{32} \; r^1_{21} \; N_1 + r^2_{43} \; r^2_{32} \; N_2 + r^3_{43} \; N_3 + N_4 \qquad (4)$$

In general, at the end of the n^{th} time period at time T_n, the profile of the building stock is

$$\begin{aligned} B_n = {} & r^1_{n,n-1} \; r^1_{n-1,n-2} \; \cdots \; r^1_{21} \; N_1 \\ & + r^2_{n,n-1} \; r^2_{n-1,n-2} \; \cdots \; r^2_{32} \; N_2 \\ & \cdots \\ & + r^{n-1}_{n,n-1} \; N_{n-1} + N_n \end{aligned} \qquad (5)$$

2. A.ROGERS, Matrix Methods of Population Analysis, *Journal of the American Institute of Planners*, V.32, n.1, Jan. 1966, pp.40-44; and A.ROGERS, *Matrix Analysis of Interregional Population Growth and Distribution*, Berkeley: University of California Press, 1968; and see also N.KEYFITZ, Matrix Multiplication as a Technique of Population Analysis, *Milbank Memorial Fund Quarterly*, XLII,4, Oct.1964, pp.68-83.

3. H.B. WOLFE, Models for Condition Aging of Residential Structures, *Journal of the American Institute of Planners*, v.33, n.7 July 1967, pp.192-195.

A MATRIX FORMULATION

The algebraic progression discussed above can best be expressed with the aid of matrix algebra. A variety of studies from different fields have adapted and utilized matrix algebra for model construction. The most common approach is the "cohort survival method" used in general population studies.[2] Few studies, among which Wolfe's can be mentioned, have incorporated matrix algebra for the analysis of the behavior of the behavior of building stock.[3]

There are a number of potential applications of matrix algebra as an analtyical tool in evaluating building stock over time which will become clearer as the model is described more fully. Briefly, however, a matrix formulation furnishes an abstract but relatively clear way of demonstrating a model. A number of computational advantages and the capacity to include a large body of information enables the matrix presentation to reduce a complex issue into a systematic form. Thus, it is a useful method of illustrating the various dimensions of building activity simultaneously. Representing changes in the building stock over time as a function of building construction and survival can be readily incorporated in a matrix formulation of the model. Moreoever, the detailed breakdown of building stock possible in the matrix formulation of this model allows for the interection of different rates of change for each cohort of buildings. This provides insights into the relation between urban processes and the physical stock over time.

The model which illustrates the successive accretion of building stock is:

$$[S] = [R][N] \qquad (6)$$

where [S] is an (nxn) matrix whose elements s_{ij} denote the number of buildings built in period j that have survived until period i. [R] is the survival rate matrix with elements R_{ij} denoting the cumulative survival rate of buildings and [N] is a diagonal matrix whose elements N_{ii} represent the number of buildings constructed during time period i.
These matrices will be discussed in detail in the following paragraphs.

Thus, the matrix [S] shows over a sequence of time periods those buildings that have survived among all that have been constructed at different points in time. Therefore, it illustrates completely the time dimension of a city's architectural inventory.

On the other hand, the size dimension of a city's physical stock my be obtained from the following matrix operation:

$$\{B\} = [S]\{u\} \tag{7}$$

where {B} is a column vector with elements B_i, each symbolizing, as before, the total number of buildings at successive time periods; and {u} is another column vector whese elements are all unit.

[R], as mentioned above, is the survival matrix with each element R_{ij} denoting the cumulative survival rate of buildings.

From the discussion given in the preceeding section

$$R_{ij} = \begin{cases} \delta_{ij} + (1-\delta_{ij}) \prod_{k=j+1}^{i} r_{k,k-1}^{(j)} & \text{for } i \geq j \\ 0 & \text{for } i < j \end{cases} \tag{8}$$

where

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

$r^{j}_{t+1,t}$ is the transition probability of survival during the time period ((t+1)-t) for buildings constructed in time period *j*.

As has been demonstrated above R_{ij} specifies the cumulative survival rate at a time period *i* of those buildings that have been constructed at an earlier time period *j*. Since *j* denotes the time of construction and *i* follows *j* in time, *i* is always greater than *j*. And for $i < j$, $R_{ij} = 0$. For $i = j$, we see that value the survival rate R_{ij} corresponding to that of the recently constructed building stock is unit. If a small increment of time is chosen, the probability of survival for the most recently constructed buildings is almost one. Therefore, a value of unity can be assumed for survival when $i = j$.[4] Consequently, the final form of the survival matrix [R] is a lower triangular matrix as shown below:

4. If a longer time period is chosen, the elements(R_{ij}) then ($i = j$) can have a value less than unity.

$$[R] = \begin{vmatrix} 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ R_{21} & 1 & 0 & \cdot & \cdot & \cdot & \cdot \\ R_{31} & R_{32} & 1 & 0 & \cdot & \cdot & \cdot \\ R_{41} & R_{42} & R_{43} & 1 & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ R_{n1} & R_{n2} & \cdot & \cdot & \cdot & R_{n,n-1} & 1 \end{vmatrix}$$

Applying equation (8) for each element, R_{ij} is obtained in the following manner:

$$R_{21} = r^1_{21}$$
$$R_{31} = r^1_{32}\, r^1_{21}$$
$$R_{41} = r^1_{43}\, r^1_{32}\, r^1_{21}$$
....
....
$$R_{n1} = r^1_{n,n-1} \cdots r^1_{43}\, r^1_{32}\, r^1_{21}$$
$$R_{32} = r^2_{32}$$
$$R_{42} = r^2_{43}\, r^2_{32}$$
....
....
$$R_{n2} = r^2_{n,n-1} \cdots r^2_{54}\, r^2_{43}\, r^2_{32}$$

Since the survival rates $r^j_{t+1,t}$ always very between zero and unity, they may be viewed as probabilities. The superscript j indicates the time period of new additions to the building stock. For example, in the equation,

$$R_{42} = r^2_{43}\, r^2_{32}$$

the superscript 2 refers to the probability of survival for buildings constructed in the second time period; r^2_{32} denotes the transition probability of survival from time T_2 to time T_3, i.e., during the 3rd time period of those structures built in the second time period; and r^2_{43} indicates the transition probability of survival from T_3 to T_4 of those structures constructed in the second time period. Therefore, the product of the two $r^2_{32}\, r^2_{43}$ is the cumulative survival rate of buildings constructed during the time period two at the end of the time period four, i.e. R_{42}.

The operational model of the city's architectural inventory in terms of the dimensions of time composition and size may thus be expressed in the following form:

A. Time Composition Dimension:

$$[S] = [R]\ [\bar{N}]$$

B. Size Dimension:

$$\{B\} = [S]\ \{u\}$$
$$= [R]\ [N]\ \{u\}$$

NUMERICAL EXAMPLE

A very simplified demonstration of this model can assist in clarifying its operation. Assume, for example, that during the initial construction period of a town, 5000 buildings are erected and no demolition occurs; and that in the next time period, only 2000 more are built, while 10% of those which were constructed in the previous period are demolished. During the third time period, the population of this area rapidly increases and 10000 new buildings are constructed. Here, however, 20% of those buildings that have survived through T_2 from the initial building stock and 10% of those constructed in the second time period are demolished. In order to determine the profile of the city's architectural inventory, the elements of the survival matrix [R] and of the diagonal matrix of the new building stock [N] are constructed for each of the three time periods.

The elements R_{ij} of the survival matrix can be calculated using equation (8). Since the demolition rates are known, survival rates can be obtained by subtracting demolition rates from unity. Hence, from equation (8)

$$R_{21} = 1 - 0.10 = 0.90$$

$$R_{31} = 0.90 \times (1-0.20) = 0.72$$

$$R_{32} = 1 - 0.10 = 0.90$$

Decomposition of the survival stock can now be obtained by the following matrix operation:

$$\begin{vmatrix} 1 & 0 & 0 \\ 0.90 & 1 & 0 \\ 0.72 & 0.90 & 1 \end{vmatrix} \times \begin{vmatrix} 5000 & 0 & 0 \\ 0 & 2000 & 0 \\ 0 & 0 & 10000 \end{vmatrix} = \begin{vmatrix} 5000 & 0 & 0 \\ 4500 & 2000 & 0 \\ 3600 & 1800 & 10000 \end{vmatrix}$$

As this formulation shows, the matrix of survival stock [S] is decomposed into a time series of survival stock. When $i = j$, the diagonal elements of [S] denote the number of buildings newly constructed at each time period. Each row in matrix [S] represents a cross-section or a profile of surviving buildings from different time periods; and the element at the far right of each row indicates the most recent construction activity. For example, at the end of the second time period, the building stock is composed of 4500 surviving buildings from those that were constructed during the first time period and an additional 2000 buildings which were recently constructed during the second time period. At the end of the final time period, 3600 buildings remain from those that were constructed during the initial time period. Eighteen hundred survive from the second time period, and, furthermore, 10000 new structures are added to the building stock. The elements in the columns of matrix [S] represent a time series of what remains from each construction period. The elements at the top of each column is the number of additions to the physical stock while the bottom element shows how many of them have survived until the final time period. In this example, of the 5000 buildings which were constructed earliest, 4500 remained in the second period, and 3600 in the third. Similarly, of those 2000 buildings constructed during the second time period, 1800 remained at the end of the most recent time period.

The total building stock at different times is also computed by a simple matrix operation using equation (7). From this equation, the size of the total building stock is found as:

$$\begin{vmatrix} 5000 & 0 & \\ 4500 & 2000 & 0 \\ 3600 & 1800 & 10\,000 \end{vmatrix} \times \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \begin{vmatrix} 5000 \\ 6500 \\ 15400 \end{vmatrix}$$

REMARKS AND CONCLUSIONS

The above described exploratory model can be decomposed into different subsets. For example, the building stock at each time period can be disaggregated by building type such that;

$$\{B\} = \{B^r\} + \{B^c\} + \{B^e\} + \ldots$$

where r denotes the residential, c the commercial and e the educational grouping of the total building stock during the initial time period. However, it would be necessary to develop a separate survival matrix for each of these because the probability that a building will exist in some subsequent period varies tremendously with the building type. Similarly, survival rates can be disaggregated due to various building materials. Another pattern which is displayed in many instances is that there can be differential survival coefficients for the interior and exterior of buildings, such as of housing stock.[5] Therefore, the matrices of the model can be disaggregated into matrix sums and vector sums with respect to different underlying factors.

The model presented above aids in systematizing our knowledge of the evolution of the city's physical stock. It assists in reconstructing the city's building stock throughout its history. This can be accomplished by gathering a variety of information pertaining to construction and demolition activity of a city or series of cities. Complete inventories do not exist, but there are a number of ways in which information can be utilized to derive estimates for the size of construction and demolition activity. Variables like population growth, population-building ratio, etc. can be used as indicators of building activity. Various analytical procedures, such as regression analysis can be integrated into the model to maintain the transition probabilities of survival and/or demolition from one subsequent time interval to another.

5. See the estimates of the useful life, depreciation rates of the structural shell of buildings and building equipment in Bulletin "F", *Income Tax, Depreciation, and Obsolescence; Estimated Useful lives and Depreciation Rates*, U.S.Treasury Department, Bureau of Internal Revenue, Washington: United States Goverment Priting Office, 1942; and, *Depreciation Guidelines an Rules*, U.S.Treasury Department, Bureau of Internal Revenue, Washington: United States Goverment Printing Office, n.32,1962.

A primary purpose in creating this kind of a model is that it establishes a structural framework to identify the problem space of decision situations on building processes and illustrate hierarchical family of related problems. A diagnosis of some of the elements of the problem space with the aid of the analytical power of the model improves the capabilities of decision making. Hence, such a model would be most beneficial during the exploratory phase of physical planning pocess. The model is intended to sensitize designers, city and preservation planners to the possible existence of variety of different building activity situations (production, survival and demolition) in any given state of development.

Given the complexity and variety of development patterns in existing urban centers and given their different distributions of building stock, the model is useful in placing an array of possible situations in an inventory form. Accordingly, each of the matrices' cross sections in time represents a specific situation in terms of the total building stock, historic remains, new building constructuon and each situation is presented in the form of survival or demolition probabilities.

In summary, the model demonstrated above in its most abstract form provides the basic organizational framework for a theory of building stock accretion. Survival continuity, a very powerful theoretical construct, assumes a key role in this model. With this model, it is possible to conceptualize physical evolution of urban centers their expansion and contraction and the significant factors that affect their survival cohorts. Different sets of events can then be described using this organizational construct. A time series analysis of the composition of cohorts allows one of to observe and understand more fully the historical transformation of the city. Furthermore, while specific trends in building activity can be obtained in a cumulative manner, the model described permits projections for changes that can be expected in the building stock in near future.

BİR YAPI STOKU ENVANTERİ

ÖZET

Bu çalışmada, kentsel merkezlerin değişmekte olan fiziksel çevresinin gelişimini imgeleyen bir model ana hatları ile aktarılmıştır. Kentin yapı stoku ögelerinin üretimi (tasarım ve yapımı), yaşamı, yıkımı ve bunları etkileyen etmenler geliştirilen bu model içinde bir bütün olarak ele alınabilmektedir. Model yapı stokunun nicel ve nitel özelliklerini içermekte, kentin yaşamında farklı zaman kesitleri içinde bu özelliklerin değişimini ortaya koyabilmektedir.

Yapı stokuna giren yeni ögeler "yapı çatkı matrisi" [N] , çatkı sonrası yaşamlarını etkileyen etmenler modelin "yapı yaşam sürdürme matrisi"ni [R] , oluşturmakta; yukarıda belirttiğimiz her iki matris [R] [N] ise kent yapı stoku bileşimlerindeki değişmeleri gösteren diğer bir matrisi, "yapı dökümü matrisi"ni [S] meydana getirmektedir. Dolayısıyla "yapı dökümü matrisi" [S], herhangi bir zamanda yapılan yapı türlerinden ne kadarının

hangi zamana veya bugüne kadar yaşamlarını sürdürebildiğini kanıtlamaktadır. Bu modelin biraz değişik bir biçimi ise bize belirli zaman kesitlerinde kentin tüm yapı stokunu vermektedir. "Tüm yapı stoku kolon vektörü" {B} "yapı dökümü matrisi" ve "birim" kolon vektörü"nden oluşmaktadır [S]{u}.

Geliştirilmiş bulunan modelde "yapı yaşam sürdürme" kavramı ağırlık taşımakta, aynı zamanda, yapı stokunun değişimini kuramsal bir yaklaşımla ele alabilmek için genel yapısal çerçeve ortaya konmaktadır. Modelin en önemli kullanımlarından birisi hiç şüphesiz çevre değerlendirmesi ve tasarımı içindir. Modelde yapı stokunun değerlendirilmesi yer ve zaman kesitleri içinde "ön" veriler dizisi şeklinde yapılabilmektedir. Model matris ögeleri çizelgesi, fiziksel çevre ve onu etkileyen etmenlerin değişik boyutlarının bir arada simgelenmesini sağlayabilmektedir. Bu çalışmada gerçekleştirilmiş bulunan model, örnekleme çalışmaları ile geliştirilecektir.

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